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TRANSFER OF HEAT BY FORCED CONVECTION FROM A LINE COMBUSTION SOURCE

DIVISION OF FIRE RESEARCH
FOREST SERVICE
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TRANSFER OF HEAT BY FORCED CONVECTION FROM A LINE COMBUSTION SOURCE--THE INFLUENCE OF ATMOSPHERIC STABILITY AND SURFACE ROUGHNESS

by

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SUMMARY

Fundamental equations of conservation of mass, momentum, and energy are applied to the problem of heat transmission by forced convection from a line combustion source. The influence of atmospheric stability and surface roughness on growth of the heated zone, decay of temperature downstream from the heat source, and aerodynamic drag is presented. Theory is general, being based on the eddy diffusivity for momentum given by Deacon (2)[1/] which is

$$K_m(z) = k\nu_\infty \left(\frac{u_* zo}{\nu_\infty}\right)\left(\frac{z}{zo}\right)^\beta$$

Eddy diffusivity for heat is taken as equal to that for momentum.

Theory presents groupings of the variables that are common to heat transfer and fluid flow problems concerning the surface air layer. Results reveal that atmospheric stability plays an important role in the growth of the heated region, decay of temperature, and aerodynamic drag for small values of friction velocity and macroviscosity parameter. For large values of these parameters (large surface roughness), atmospheric stability is not important.

1/ For underlined numbers in parentheses, refer to literature cited, page 40.

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INTRODUCTION

Combustion of a fuel generates heat which must be transferred. Transmission of thermal energy is by the conventional modes of heat transfer--conduction, free and forced convection, and radiation. A line fire burning with a moderate wind speed in light fuels, such as grass, the heat transfer is predominately by forced convection. On such a fire as well as on some back fires, free convection plays a minor role since the momentum transfer by the buoyant forces is small compared to the momentum transfer by the inertia forces existing in the velocity field. Conduction to the ground is negligible. Flame radiation which depends upon flame characteristics and geometry is of greater importance. Experiments were conducted by the Forest Products Research Laboratory in England (9) on the combustion of crib fires with wood as a fuel. Heat balances made on the combustion process indicated that heat transferred by convection was 2 to 4 times greater than by radiation. If the flame geometry is small, implying moderate rates of energy release, the radiation exchange may be neglected. On the basis of the foregoing postulates, the convection zone may be studied by using heat transfer and fluid flow theories.

In a study of the convection zone generated by a combustion process, the combustion mechanism may be neglected. Heat generated by combustion of the fuel is assumed to be known and from the theoretical viewpoint represents the heat source strength. Theory is based upon conservation of mass, momentum, and energy which is amenable to mathematical analysis. Despite its complexity, the combustion process is itself of theoretical interest. To understand a fire burning in the atmosphere, the steady-state combustion mechanism must be studied. Ultimately, the transient combustion process requires investigation. Since the power of any combustion process, whether it be for constructive or destructive purposes, is determined at the combustion zone, this region is of prime importance. It is not the purpose of this report to discuss combustion theory but only to stress its importance and direct relation to heat transfer. The combustion process generates sensible heat which must undergo transmission. By assuming the combustion zone as a line source of heat, transfer of heat by forced convection may be analyzed. A generalized approach is presented investigating forced convection heat transfer over rough surfaces as influenced by atmospheric stability.

Prior to analysis of the specific problem, the fundamental equations are presented including the buoyancy term in the momentum equation. For a given heat source strength, the minimum magnitude of wind velocity is derived so that the buoyancy term may be neglected. When this condition is imposed a solution of the system of equations is possible.

Thereafter the system of partial differential equations is transformed to a system of ordinary differential equations. Solutions with appropriate boundary conditions are obtained using the differential analyzer. Growth of the heated zone may be determined as a function of

friction velocity ratio, macroviscosity parameter, Reynolds Number, and atmospheric stability. Temperature profiles and decay of profiles downstream from the heat source as a function of velocity ratio, macroviscosity parameter, Reynolds Number, and atmospheric stability are presented. The heat source is expressed in terms of the combustion process. Finally, the shear stress and drag coefficient are determined.

Results of the investigation are applicable to line fires of moderate heat source strength wherein forced convection is the dominating mode of transfer. Likewise, the results are of interest in the diffusion of mass from a line source which is of importance in chemical warfare, smoke screen operations, and disposal of industrial plant waste gases.

ANALYSIS

Basic Equations

The continuity, momentum, and energy equations for the two-dimensional flow due to a line source for a fluid of constant Prandtl Number, specific heat, and viscosity are

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho w \frac{\partial u}{\partial z} = - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) \tag{2}$$

$$\rho u \frac{\partial w}{\partial x} + \rho w \frac{\partial w}{\partial z} = - \frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) - \rho g \tag{3}$$

$$\rho g J c_p u \frac{\partial T}{\partial x} + \rho g J c_p w \frac{\partial T}{\partial z} = J k' \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right) \tag{4}$$

nonsense

In most meteorological problems the pressure change in the direction of the mean wind is small, especially within the surface boundary layer region. This term is small compared to the other terms and may be neglected.

Applying equation (3) in the region exterior to the zone of influence states

$$\frac{\partial p_\infty}{\partial z} = - \rho_\infty g$$

The foregoing equations become

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{5}$$

$$\rho u \frac{\partial u}{\partial x} + \rho w \frac{\partial u}{\partial z} = \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) \qquad (6)$$

nonsense

$$\rho u \frac{\partial w}{\partial x} + \rho w \frac{\partial w}{\partial z} = - \frac{\partial (p - p_\infty)}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) + g(\rho_\infty - \rho) \qquad (7)$$

$$\rho g J c_p u \frac{\partial T}{\partial x} + \rho g J c_p w \frac{\partial T}{\partial z} = J k' \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) \qquad (8)$$

From the perfect gas law at constant pressure

$$\frac{\rho_\infty}{\rho} = \frac{T}{T_\infty}$$

there results

$$\rho_\infty - \rho = \rho \left(\frac{T - T_\infty}{T_\infty}\right)$$

The foregoing equations become

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \qquad (9)$$

$$u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) \qquad (10)$$

$$u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} = - \frac{1}{\rho} \frac{\partial (p - p_\infty)}{\partial z} + \frac{\mu}{\rho} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) + g \left(\frac{T - T_\infty}{T_\infty}\right) \qquad (11)$$

$$u \frac{\partial (T - T_\infty)}{\partial x} + w \frac{\partial (T - T_\infty)}{\partial z} = \frac{\nu}{\sigma} \left[\frac{\partial^2 (T - T_\infty)}{\partial x^2} + \frac{\partial^2 (T - T_\infty)}{\partial z^2}\right] \qquad (12)$$

In addition, the relation stating that the heat output of the source is accounted for at any cross-section in the region of influence yields

$$\frac{Q}{L} = \int_0^\infty \rho_\infty g c_p u \, (T - T_\infty) \, dz \qquad (13)$$

The theory may now be expressed in nondimensional form. If u_∞ is a typical velocity and x_o a typical length, then $\frac{u_\infty x_o}{\nu_\infty}$ becomes a nondimensional parameter. Writing

$$X = \frac{x}{x_o} \qquad Z = \left(\frac{u_\infty x_o}{\nu_\infty}\right)^{1/2} \frac{z}{x_o}$$

$$U = \frac{u}{u_\infty} \qquad W = \left(\frac{u_\infty x_o}{\nu_\infty}\right)^{1/2} \frac{w}{u_\infty}$$

$$P = \frac{p - p_\infty}{\rho_\infty u_\infty^2} \qquad G = \frac{(T - T_\infty)\, L\nu_\infty \rho_\infty g c_p}{Q} \left(\frac{u_\infty x}{\nu_\infty}\right)^{1/2}$$

The boundary conditions are

$$z = 0 \qquad u = w = 0 \qquad \frac{\partial T}{\partial z} = 0 \qquad \frac{\partial G}{\partial z} = 0$$

$$z \longrightarrow \infty \qquad u = u_\infty \qquad T = T_\infty \qquad G = 0$$

Upon substitution the equations become

$$\frac{\partial U}{\partial X} + \frac{\partial W}{\partial Z} = 0$$

$$U \frac{\partial U}{\partial X} + W \frac{\partial U}{\partial Z} = \frac{1}{\left(\frac{u_\infty x_o}{\nu_\infty}\right)} \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Z^2}$$

$$\frac{1}{\left(\frac{u_\infty x_o}{\nu_\infty}\right)} U\left[\frac{\partial W}{\partial X} + W \frac{\partial W}{\partial Z}\right] = - \frac{\partial P}{\partial Z} + \frac{1}{\left(\frac{u_\infty x_o}{\nu_\infty}\right)^2} \frac{\partial^2 W}{\partial X^2} + \frac{1}{\left(\frac{u_\infty x_o}{\nu_\infty}\right)} \frac{\partial^2 W}{\partial Z^2}$$

$$+ G \frac{Q}{L\rho_\infty c_p T_\infty u_\infty^3}$$

$$U \frac{\partial G}{\partial X} + W \frac{\partial G}{\partial Z} = \frac{1}{\sigma} \frac{1}{\left(\frac{u_\infty x_o}{\nu_\infty}\right)} \frac{\partial^2 G}{\partial X^2} + \frac{1}{\sigma} \frac{\partial^2 G}{\partial Z^2}$$

$$\int_o^\infty UGdZ = 1$$

On the assumption that the derivatives which occur remain finite when $\frac{u_\infty x_o}{\nu_\infty} \gg 1$, the limiting form of these equations is

$$\frac{\partial U}{\partial X} + \frac{\partial W}{\partial Z} = 0 \tag{14}$$

$$U \frac{\partial U}{\partial X} + W \frac{\partial U}{\partial Z} = \frac{\partial^2 U}{\partial Z^2} \tag{15}$$

$$-\frac{\partial P}{\partial Z} = G \frac{Q}{L\rho_\infty c_p T_\infty u_\infty^3} \tag{16}$$

$$U \frac{\partial G}{\partial X} + W \frac{\partial G}{\partial Z} = \frac{1}{\sigma} \frac{\partial^2 G}{\partial Z^2} \tag{17}$$

$$\int_o^\infty UGdZ = 1 \tag{18}$$

From equation (16), restriction to a boundary-layer type problem which neglects the buoyancy term is given by

$$\frac{Q}{L} \ll \rho_\infty c_p T_\infty u_\infty^3$$

The function G is of order unity, having a value of zero at the exterior region and 0.66 at the surface from equation (34). Property values of air to be used throughout this paper are

$\nu_\infty = 1.80 \times 10^{-4}$ sq.ft/sec.

$\rho_\infty = 2.34 \times 10^{-3}$ lb.sec.^{2}/ft.^{4}

$c_p = 0.24$ BTU/lb. ^oR

$T_\infty = 530^oR$

Using the foregoing values the relation is

$$\frac{Q}{L} \ll 0.30\ u_\infty^3$$

When the term is arbitrarily reduced to 10 percent of the preceding value, that is, letting $Q/L = 0.03\ u_\infty^3$, the relation is shown in figure 1. For a known heat output, the velocity must be equal to or greater than the value given by figure 1. Buoyancy may then be neglected.

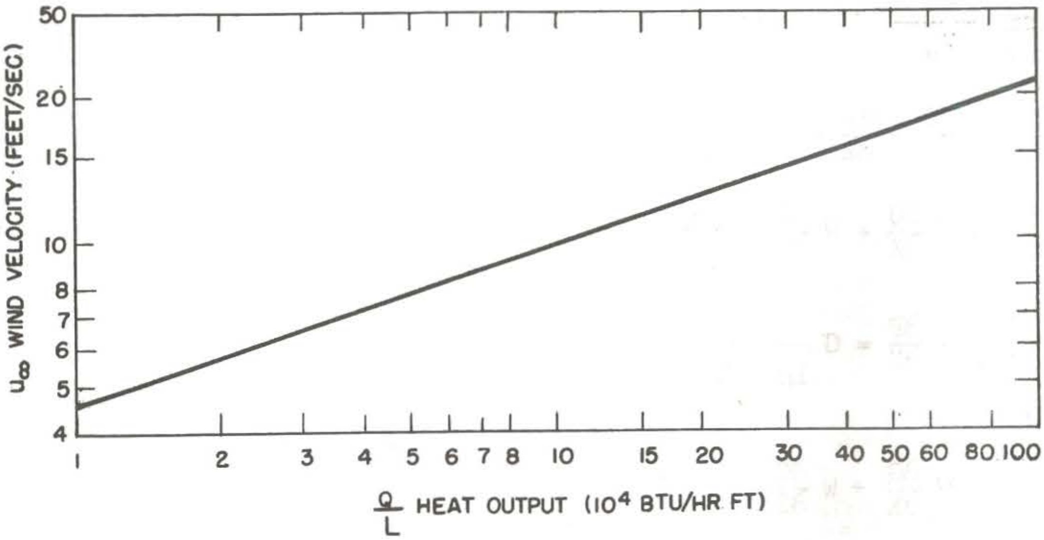

Figure 1.--Minimum velocity for given heat output to neglect buoyancy

The relation has been calculated for a line combustion of a forest fuel of rate R tons per acre per minute and width of flame front L*. This is shown in figure 2.

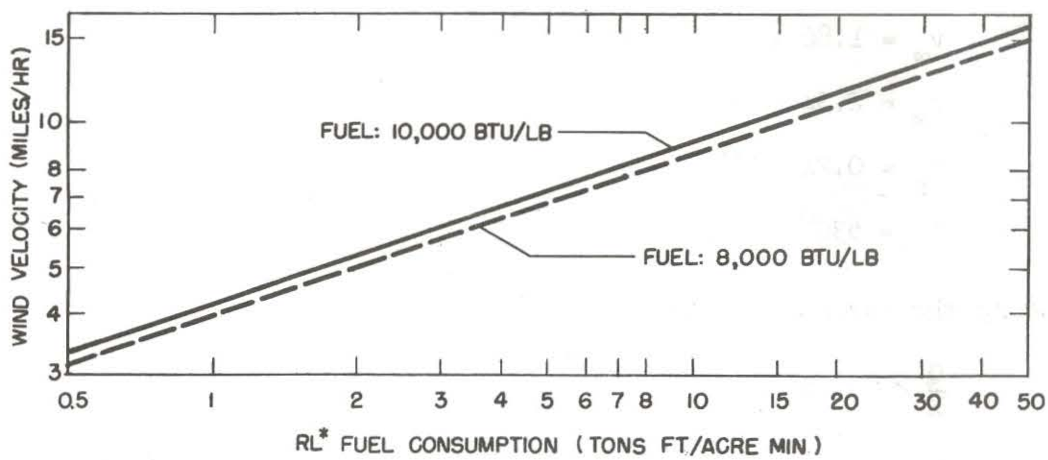

Figure 2.--Minimum wind velocity for given fuel consumption to neglect buoyancy

The meaning of the relationship shown in Figure 1 (or the relationship in Figure 2) can be illustrated by using it to compute specific rates of spread corresponding to a given wind velocity for which buoyancy can be neglected. For example, in a cover type with a light fuel[2/] of 0.5 tons per acre in which a fire is spreading at the rate of 30 feet per minute, the wind speed would have to be 14 feet per second or greater in order that buoyancy could be neglected. For this same light fuel, 0.5 tons per acre, but with a rate of spread of 3 feet per minute, the wind speed would have to be 6.5 feet per second or greater for buoyancy to be neglected. However, if the wind speed were 6.5 feet per second for a considerably heavier fuel, 15 tons per acre, then the rate of spread of a fire in this fuel would have to be 0.1 feet per minute or less for buoyancy to be neglected.

The condition for neglecting buoyant forces has been established in the preceding section. To apply equations (9) to (12) inclusive to a transport process in an atmosphere it is necessary to introduce variable diffusivities of momentum and heat $K_m(z)$ and $K_H(z)$. The equations (9), (10), and (12) in dimensional form then become

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{19}$$

$$u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = \frac{\partial}{\partial z} \left[K_m(z) \frac{\partial u}{\partial z} \right] \tag{20}$$

$$u \frac{\partial (T - T_\infty)}{\partial z} + w \frac{\partial (T - T_\infty)}{\partial z} = \frac{\partial}{\partial z} \left[K_H(z) \frac{\partial (T - T_\infty)}{\partial z} \right] \tag{21}$$

with boundary conditions

$$z = 0 \qquad u = w = 0 \qquad \frac{\partial (T - T_\infty)}{\partial z} = 0$$

$$z \longrightarrow \infty \quad u = u_\infty \qquad T - T_\infty = 0$$

Deacon (2) studied the vertical diffusion in the lowest layers of the atmosphere. Field experiments were conducted over two types of natural land surfaces. From the measurements, the vertical component of the eddy viscosity was determined for the lowest 30 meters of the atmosphere. The result was

$$K_m(z) = k\, \nu_\infty \left(\frac{u_* z_o}{\nu_\infty} \right) \left(\frac{z}{z_o} \right)^\beta \tag{22}$$

2/ Computation based on a heat value of fuel of 8,000 BTU per pound.

Rider (3) conducted an experimental study of the factors which influence the vertical turbulent transport of momentum, water vapor, and heat in the first two meters of air above a short grass surface. Measurements were made of the vertical profiles of velocity, humidity, and temperature, evaporation rate, and aerodynamic drag of the surface. An attempt was made to measure the individual terms involved in the heat balance.

Results indicated that the wind velocity and temperature profiles always have the same distribution, and this distribution is representative of the humidity profile in the majority of cases. The surface friction coefficient was found to be independent of wind speed, increasing in magnitude during unstable conditions. Values of the eddy diffusivity for momentum, water vapor, and heat were obtained by measuring the quantities in the expressions defining the diffusivities. Results of this investigation showed that the diffusivities for momentum and vapor are identical over the range of stability of the experimental investigation. The value of the diffusivity for heat was approximately the same as for momentum and vapor, except that on occasion it is much larger than the other two.

Despite limited experimental evidence, expressions for the eddy diffusivities for momentum and heat were taken as equal. Thus

$$K_H(z) = k\,\nu_\infty \left(\frac{u_* z_o}{\nu_\infty}\right)\left(\frac{z}{z_o}\right)^{\beta} \tag{23}$$

The set of partial differential equations can be greatly simplified by transformation to a new coordinate system. Introduction of the stream function ψ automatically satisfies the continuity equation. The velocity components are given by

$$u = \frac{\partial \psi}{\partial z} \qquad\qquad w = -\frac{\partial \psi}{\partial x} \tag{24}$$

The formulas for making the transformations from the independent variables x, z to an arbitrary and completely new set x, η are

$$\begin{aligned} \eta &= \frac{u_\infty}{\nu_\infty}\, k^{-\frac{1}{2-\beta}} \left(\frac{u_*}{u_\infty}\right)^{-\frac{\beta}{2-\beta}} \left(\frac{u_* z_o}{\nu_\infty}\right)^{-\frac{1-\beta}{2-\beta}} \left(\frac{u_\infty x}{\nu_\infty}\right)^{-\frac{1}{2-\beta}} z \\ \psi &= k^{\frac{1}{2-\beta}}\, \nu_\infty \left(\frac{u_*}{u_\infty}\right)^{\frac{\beta}{2-\beta}} \left(\frac{u_* z_o}{\nu_\infty}\right)^{\frac{1-\beta}{2-\beta}} \left(\frac{u_\infty x}{\nu_\infty}\right)^{\frac{1}{2-\beta}} f(\eta) \end{aligned} \tag{25}$$

After transformation

$$u = u_\infty \frac{\partial f}{\partial \eta}$$

$$w = \left(\frac{1}{2-\beta}\right) k^{\frac{1}{2-\beta}} \nu_\infty \left(\frac{u_*}{u_\infty}\right)^{\frac{\beta}{2-\beta}} \left(\frac{u_* z_o}{\nu_\infty}\right)^{\frac{1-\beta}{2-\beta}} \left(\frac{u_\infty x}{\nu_\infty}\right)^{\frac{1}{2-\beta}} \frac{1}{x}\left(\eta \frac{\partial f}{\partial \eta} - f\right) \qquad (26)$$

$$T - T_\infty = k^{-\frac{1}{2-\beta}} \frac{Q}{L\rho_\infty g c_p \nu_\infty} \left(\frac{u_*}{u_\infty}\right)^{-\frac{\beta}{2-\beta}} \left(\frac{u_* z_o}{\nu_\infty}\right)^{-\left(\frac{1-\beta}{2-\beta}\right)} \left(\frac{u_\infty x}{\nu_\infty}\right)^{-\frac{1}{2-\beta}} g(\eta)$$

and equations (20) and (21) become

$$\frac{d}{d\eta}\left(\eta^\beta \frac{d^2 f}{d\eta^2}\right) + \left(\frac{1}{2-\beta}\right) f \frac{d^2 f}{d\eta^2} = 0 \qquad (27a)$$

$$\frac{d}{d\eta}\left(\eta^\beta \frac{dg}{d\eta}\right) + \left(\frac{1}{2-\beta}\right) \frac{d(fg)}{d\eta} = 0 \qquad (27b)$$

with boundary conditions

$\eta = 0$	$f = f' = 0$	$g' = 0$
$\eta \longrightarrow \infty$	$f' = 1$	$g = 0$

Conservation of energy at any cross-section downstream from the heat source yields

$$\int_o^\infty \rho_\infty g c_p u \, (T - T_\infty) \, dz = Q/L \qquad (28)$$

which is transformed to

$$\int_o^\infty \frac{df}{d\eta} g \, d\eta = 1 \qquad (29)$$

Laminar Solution

The laminar flow case is a special case of the more generalized system and is presented here in the interest of completeness. If $\beta = 0$ and $k\left(\frac{u_* z_o}{\nu_\infty}\right)$ equals unity, then the eddy viscosity relation is that for

laminar flow ($\sigma = 1$)

$$K_m(z) = K_H(z) = \nu_\infty$$

The equations (27a) and (27b) become

$$\frac{d^3f}{d\eta^3} + \frac{1}{2} f \frac{d^2f}{d\eta^2} = 0 \tag{30a}$$

$$\frac{d^2g}{d\eta^2} + \frac{1}{2} \frac{d(fg)}{d\eta} = 0 \tag{30b}$$

Equation (30a) has been solved for the boundary conditions of this problem and is presented in Table III of reference (5). Solution of equation (30b) is

$$g(\eta) = Ke^{-\frac{1}{2}\int_o^\eta f \, d\eta} \tag{31}$$

where K is given by equation (29) as

$$K = \frac{1}{\int_o^\infty \frac{df}{d\eta} e^{-\frac{1}{2}\int_o^\eta f \, d\eta} d\eta} = 0.66 \tag{32}$$

By selecting the edge of the boundary layer as the position where the velocity attains 99 percent of the value of u_∞, there results $\eta_\delta = 5.0$ which gives

$$\frac{\delta u_\infty}{\nu_\infty} = 5.0 \left(\frac{u_\infty x}{\nu_\infty}\right)^{1/2} \tag{33}$$

The temperature at the surface downstream from the heat source is

$$T_w - T_\infty = 0.66 \frac{Q}{L\rho_\infty g c_p \nu_\infty} \left(\frac{u_\infty x}{\nu_\infty}\right)^{-\frac{1}{2}} \tag{34}$$

The laminar flow result states that the temperature is linearly dependent upon the heat source strength and decays inversely as the square root of the velocity and distance from the source.

Turbulent Solution

Transfer processes in the atmosphere are not governed by laminar flow theory. In the foundation of the theory, the diffusion mechanism applicable to the atmosphere was used.

Solution of the general system, equations (27a) and (27b), was undertaken by the University of California at Los Angeles, Department of Engineering Differential Analyzer.[3]

Integration of equation (27a) gives

$$\eta^{\beta}\,\frac{d^2f}{d\eta^2} + \left(\frac{1}{2-\beta}\right)\int_{\lambda=\epsilon}^{\eta} f\,\frac{d^2f}{d\lambda^2}\,d\lambda = \epsilon^{\beta}\,\frac{d^2f}{d\eta^2}\bigg|_{\eta=\epsilon} \tag{35}$$

Integration is started a small distance ϵ from the origin due to the singularity at $\eta \equiv 0$. The value of the right side of equation (35) is defined as

$$\epsilon^{\beta}\,\frac{d^2f}{d\eta^2}\bigg|_{\eta=\epsilon} = K_{1} \tag{36}$$

In order to eliminate the two point boundary condition thereby facilitating machine solution the transformation

$$F = f/A \quad \text{and} \quad N = \eta/B$$

is introduced into equation (35) and choosing

$$A = (2-\beta)B^{\beta-1} \quad \text{and} \quad K_{1} = AB^{\beta-2}$$

there results on integration

$$\frac{dF}{dN} = \int_{\epsilon/B}^{N} \left[1 - \int_{\epsilon/B}^{N} F\, d\left(\frac{dF}{dN}\right)\right]\frac{dN}{N^{\beta}} + F'\left(\frac{\epsilon}{B}\right) \tag{37}$$

with boundary conditions

$$N = \epsilon/B \qquad F \text{ and } F' \text{ finite}$$

$$\frac{dF}{dN}\bigg|_{N \longrightarrow \infty} = B/A$$

3/ Messrs. Malcolm R. Davis and Don Lebell are responsible for the mathematical steps which lead to setting up the equations for the differential analyzer including the trial and error method.

To begin the machine solution, the function f' is expanded in a Taylor series about the point ϵ.

$$f'(\eta) = f'(\epsilon) + f''(\epsilon)\,(\eta-\epsilon) + \frac{f'''(\epsilon)}{2!}\,(\eta-\epsilon)^2 + \ldots\ldots$$
$$+ \frac{f^{n+1}(\epsilon)}{n!}\,(\eta-\epsilon)^n + \ldots.$$

which for $\eta = 0$ yields

$$f'(0) = 0 = f'(\epsilon) - f''(\epsilon)\epsilon + \frac{f'''(\epsilon)}{2!}\,\epsilon^2 + \ldots\ldots$$
$$+ (-1)^n \frac{f^{n+1}(\epsilon)}{n!}\,\epsilon^n + \ldots..$$

or

$$0 = \frac{f'(\epsilon)}{\epsilon} - f''(\epsilon) + \frac{f'''(\epsilon)}{2!}\,\epsilon + \ldots\ldots.$$
$$+ (-1)^n \frac{f^{n+1}(\epsilon)}{n!}\,\epsilon^{n-1} + \ldots\ldots$$

If the higher order derivatives are assumed finite and ϵ is selected arbitrarily small, then the higher order terms may be neglected so that

$$\frac{f'(\epsilon)}{\epsilon} = f''(\epsilon) \tag{38}$$

A value of ϵ/B is arbitrarily selected with zero initial values of F and F'. A run with the differential analyzer is completed which determines A, B, and K_1. From these results ϵ may be calculated and $f(\epsilon)$ is determined by equation (36). Successively smaller values of ϵ/B, are assumed and machine solutions obtained until the calculated $f(\epsilon)$ when inserted in equation (38) yields an $f(\epsilon)$ which is small compared to $f(\infty) = 1$. The final assumed value of ϵ/B and calculated value of $(f(\epsilon)\,B/A)$ with F as zero is run to see that the behavior of F is reproduced.

The equation (27b) is transformed and integrated as

$$\ln G = -\int_{\epsilon/B}^{N} \frac{F}{N^{\beta}}\,dN \tag{39}$$

where $G = \dfrac{g(\eta)}{g(\epsilon)}$

The differential analyzer run was performed for three values of β , namely, 3/4, 1, and 5/4 which represent subadiabatic, neutral, and superadiabatic conditions.

The condition of conservation of energy given by equation (29) is transformed to

$$g(0) = \frac{1}{A\int_0^\infty \frac{dF}{dN} G dN}$$

Results of the integration are the functions f and $\frac{df}{d\eta}$ which are presented in figures 3 and 4. The function G is presented in figure 5. Also revealed in figure 6 is the integrand which appears in the conservation of energy relation.

RESULTS

Velocity Ratio and Macroviscosity Parameter

Theoretical investigation of heat transfer from a line source in the atmosphere yields the effect of friction velocity, macroviscosity parameter, and atmospheric stability on the growth of the temperature zone. For a given surface and magnitude of wind velocity the effect of atmospheric stability on the heat transfer may be determined. This effect is due to the variation of the vertical component of eddy diffusivity as the stability changes. Results are presented using values of friction velocity and macroviscosity given by Sheppard (6). The friction velocity is defined by the relation

$$u_*^2 = \left|\frac{\tau}{\rho_\infty}\right|$$

Near the surface the shearing stress is considered constant and equal to the value at the surface. The friction velocity is then

$$u_* = \left(\frac{\tau}{\rho_\infty}\right)^{1/2}$$

and depends on the nature of the surface and the magnitude of the mean velocity. The macroviscosity parameter is considered as a quantity which plays a role in fully rough flow analogous to that experienced by the kinematic viscosity in smooth flow.

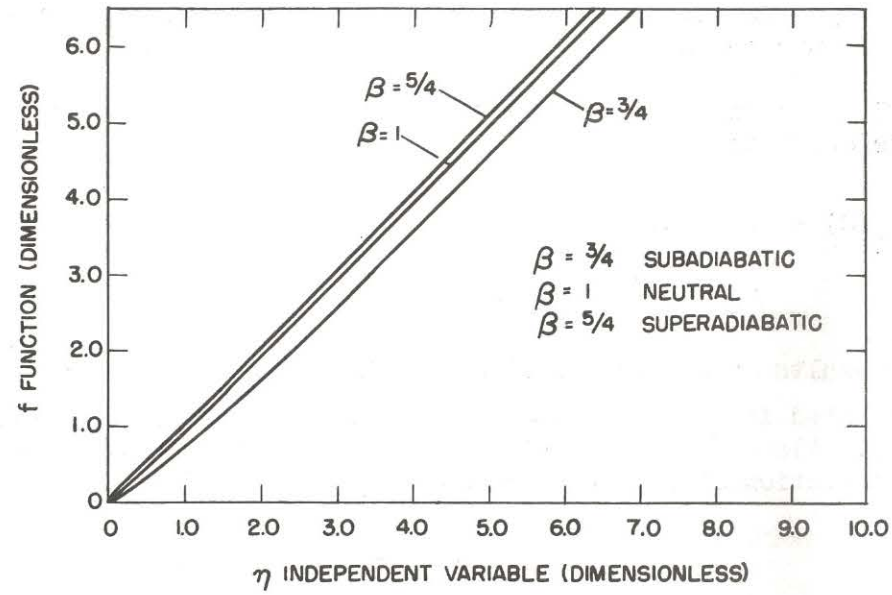

Figure 3.--Variation of function with independent variable as dependent on stability

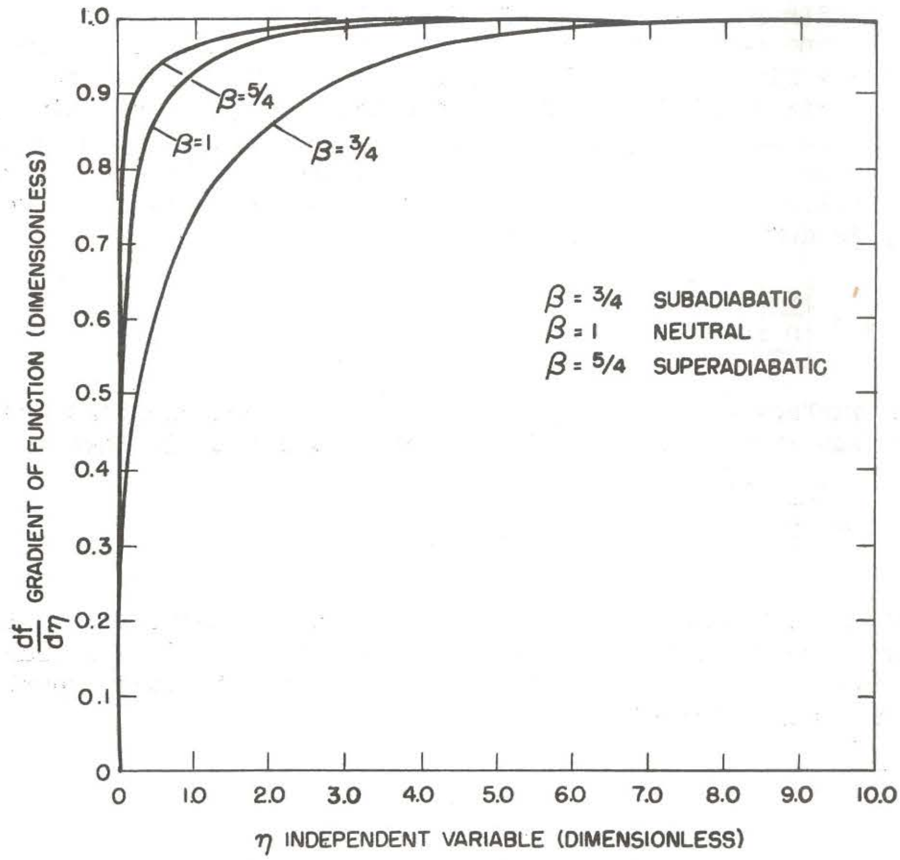

Figure 4.--Variation of gradient with independent variable as dependent on stability

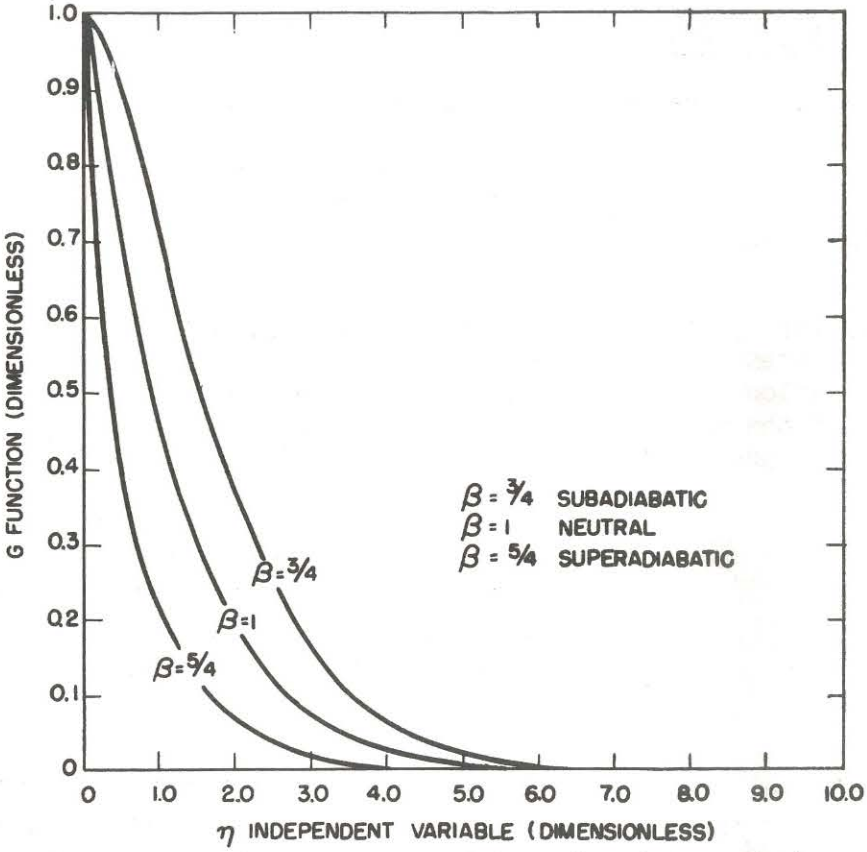

Figure 5.--Variation of function with independent variable as dependent on stability

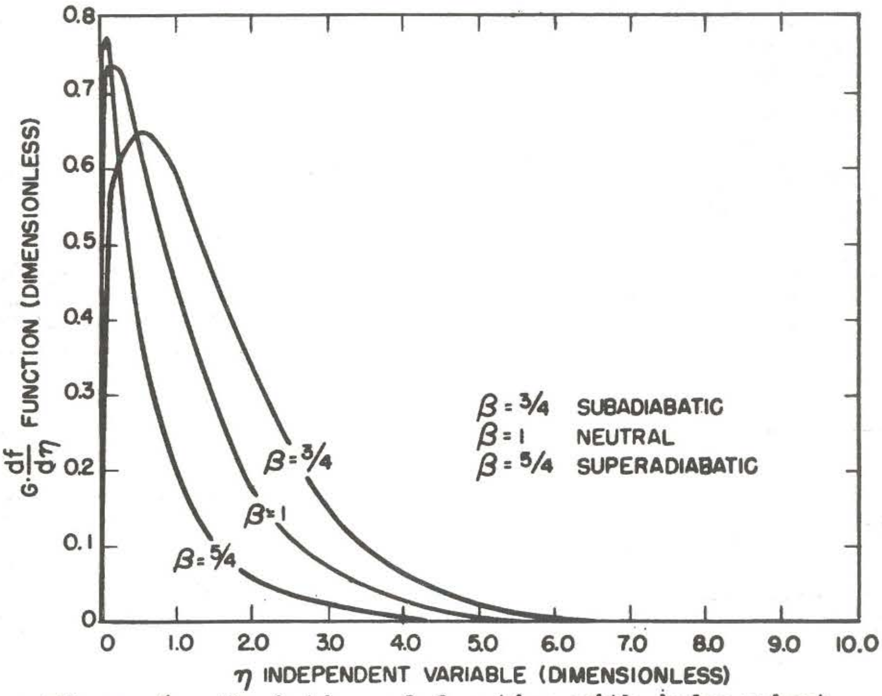

Figure 6.--Variation of function with independent variable as dependent on stability

Growth of Heated Zone

Growth of the heated zone is given by the generalized equation

$$\frac{u_\infty \delta}{\nu_\infty} = \eta_\delta \, k^{\frac{1}{2-\beta}} \left(\frac{u_*}{u_\infty}\right)^{\frac{\beta}{2-\beta}} \left(\frac{u_* z_o}{\nu_\infty}\right)^{\frac{1-\beta}{2-\beta}} \left(\frac{u_\infty x}{\nu_\infty}\right)^{\frac{1}{2-\beta}} \tag{40}$$

The parameter β is dependent on the stability of the atmosphere and has been correlated with the Richardson Number by Deacon (2). Figure 7 is the correlation taken from Deacon. Values of η_δ are determined from solution of the differential equation for the three values of β. These values are presented in table 1.

Table 1.--Variation of η_δ with β

β	η_δ
0.75	7.10
1.00	6.67
1.25	5.82

Application of the theory requires specification of the Richardson Number. The stability parameter β is then obtained from figure 7. Figure 8 yields the required magnitude of η_δ. For the natural surface considered, the value of the dimensionless friction velocity ratio u_*/u_∞ and the macroviscosity parameter $u_* z_o/\nu_\infty$ are available in the literature or must be determined experimentally. Growth of the region of influence δ may be calculated as a function of distance from the source x for the specified wind velocity u_∞.

Sheppard (6) published values of friction velocity ratio and macroviscosity parameter. These are presented in table 2.

Table 2.--Representative values of u_*/u_∞ and $u_* z_o/\nu_\infty$ for natural surfaces. Neutral stability u_* corresponds to u_∞ = 500 cm/sec at 200 cm

	u_*/u_∞	$u_* z_o/\nu_\infty$
Very smooth (mud flats, ice)	0.0316	0.096
Lawn, grass up to 1 cm high	0.050	15.6
Downland, thin grass up to 10 cm high	0.071	156.0
Thick grass, up to 10 cm high	0.089	623.0
Thin grass, up to 50 cm high	0.107	1647.0
Thick grass, up to 50 cm high	0.127	3353.0

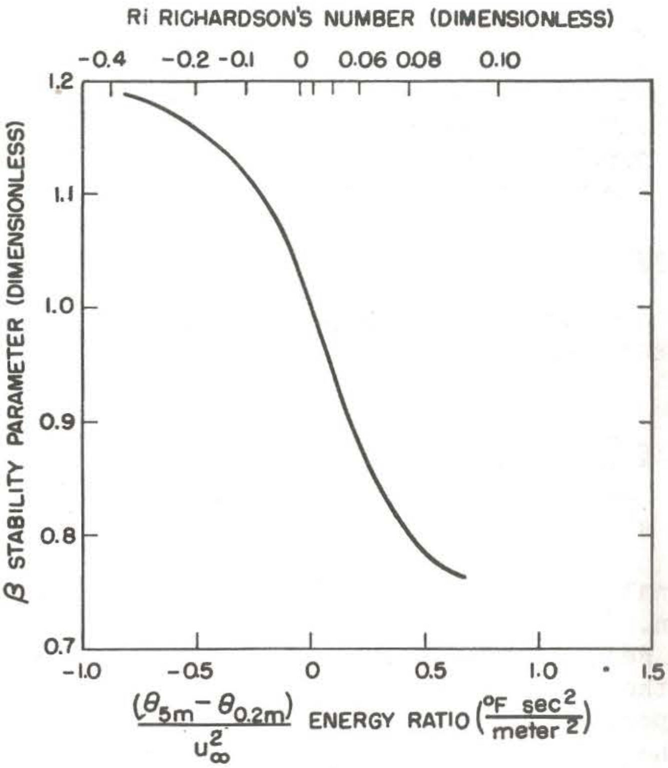

Figure 7.--Variation of β with stability (after Deacon)

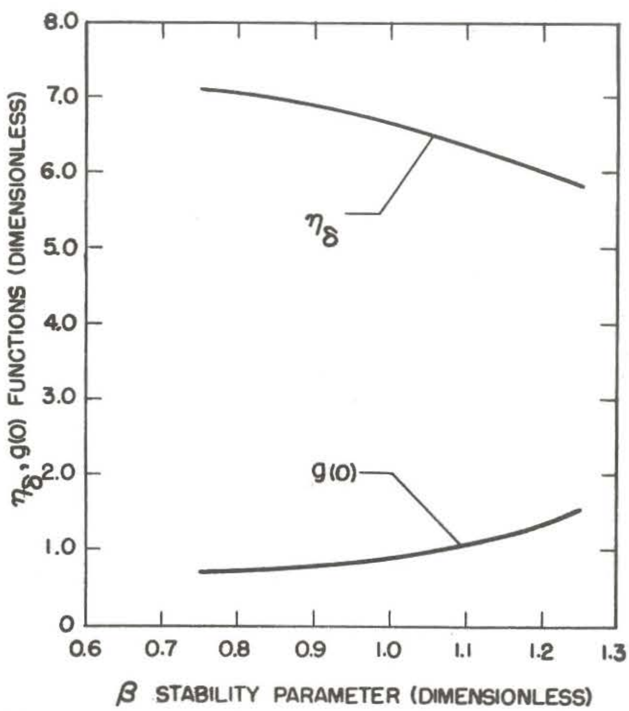

Figure 8.--Variation of functions η_δ and $g(0)$ with stability

Theoretical results are presented in figure 9 which reveals growth of the heated zone. Natural surfaces of lawn grass up to 1 cm high and thick grass up to 50 cm high are compared. A wind velocity of 17 ft/sec was arbitrarily selected. For these conditions the effect of stability on growth of the heated zone is evident. The generalized correlation for the two surfaces is presented in figure 10.

Temperature Decay

Decay of temperature downstream from the heat source is given by the generalized equation

$$T - T_\infty = \frac{g(0)}{k^{\frac{1}{2-\beta}}} \frac{Q}{L\rho_\infty g c_p \nu_\infty} \left(\frac{u_*}{u_\infty}\right)^{-\frac{\beta}{2-\beta}} \left(\frac{u_* z_o}{\nu_\infty}\right)^{-\left(\frac{1-\beta}{2-\beta}\right)} \left(\frac{u_\infty x}{\nu_\infty}\right)^{-\frac{1}{2-\beta}} G(\eta) \qquad (41)$$

The nondimensional velocity ratio and macroviscosity parameter also appear in this equation. Strength of the heat source must be specified. The function $G(\eta)$ results from the solution of the differential equation and represents the generalized temperature profile in the heated layer. The surface temperature is maximum and is denoted by $G(\eta) = 1$. Values of $g(0)$ for the three cases of β are presented in table 3.

Table 3.--Variation of $g(0)$ with β

β	$g(0)$
0.75	0.70
1.00	0.94
1.25	1.58

This function is also shown in figure 8. Specification of the parameter gives the surface temperature as a function of distance downstream from the heated source for a specified wind velocity. Influence of the parameters on decay of surface temperature is presented in figure 11. Natural surfaces of lawn grass up to 1 cm high and thick grass up to 50 cm high were selected. A wind velocity of 17 ft/sec and heat output of 60 BTU/sec ft were arbitrarily selected. This heat output corresponds to a forest fuel rate of 10 ton ft/acre min. based upon a 100% combustion efficiency considering forced convection only. This fuel rate is 21.6 x 10^4 BTU/hr ft and figure 1 indicates that a wind velocity of 13 ft/sec or greater is required permitting the neglect of the buoyancy effect. The influence of stability on decay of surface temperature is calculated. Generalized correlations for these two natural surfaces are presented in figure 12.

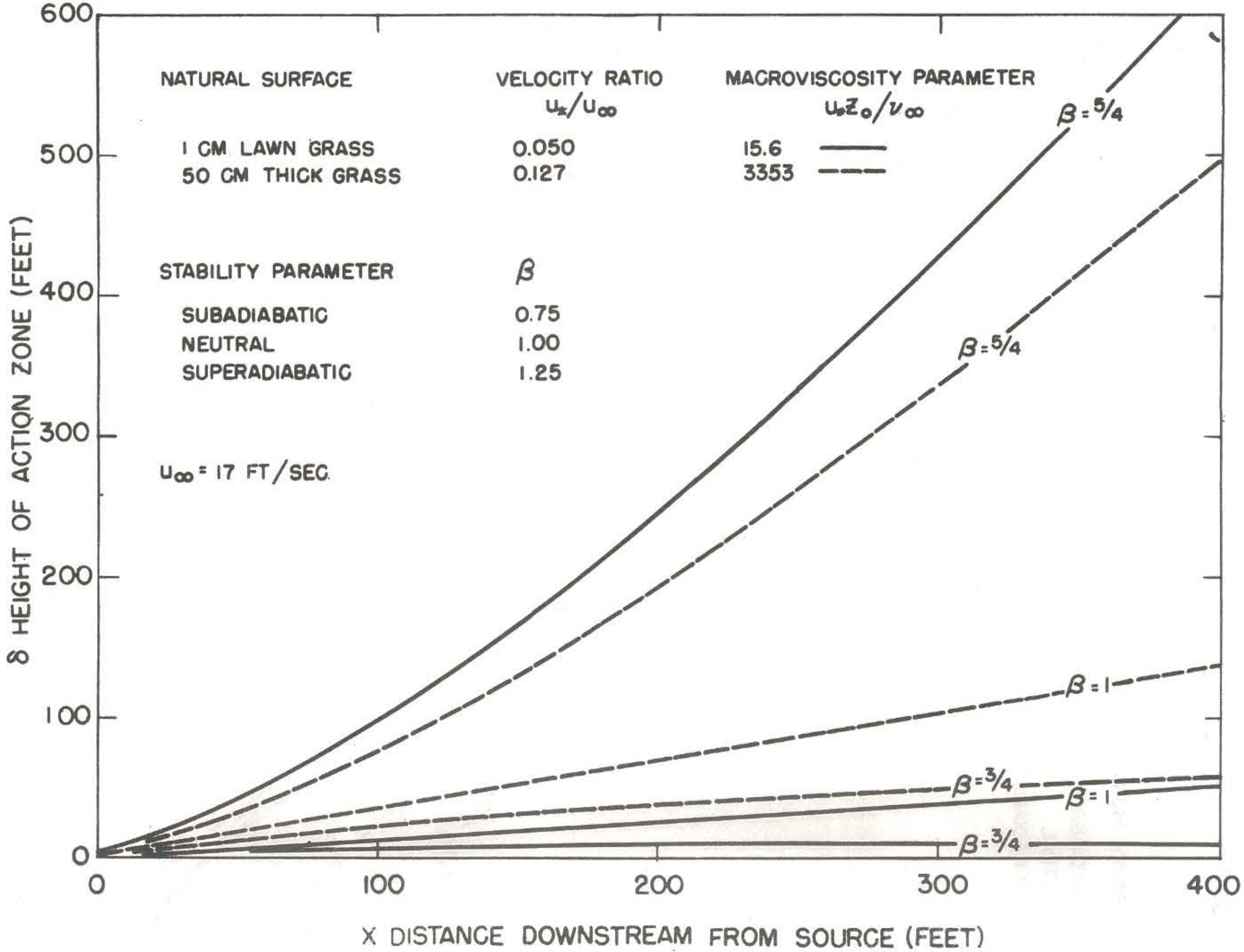

Figure 9.--Growth of action zone as function of velocity ratio, macroviscosity parameter, and stability parameter

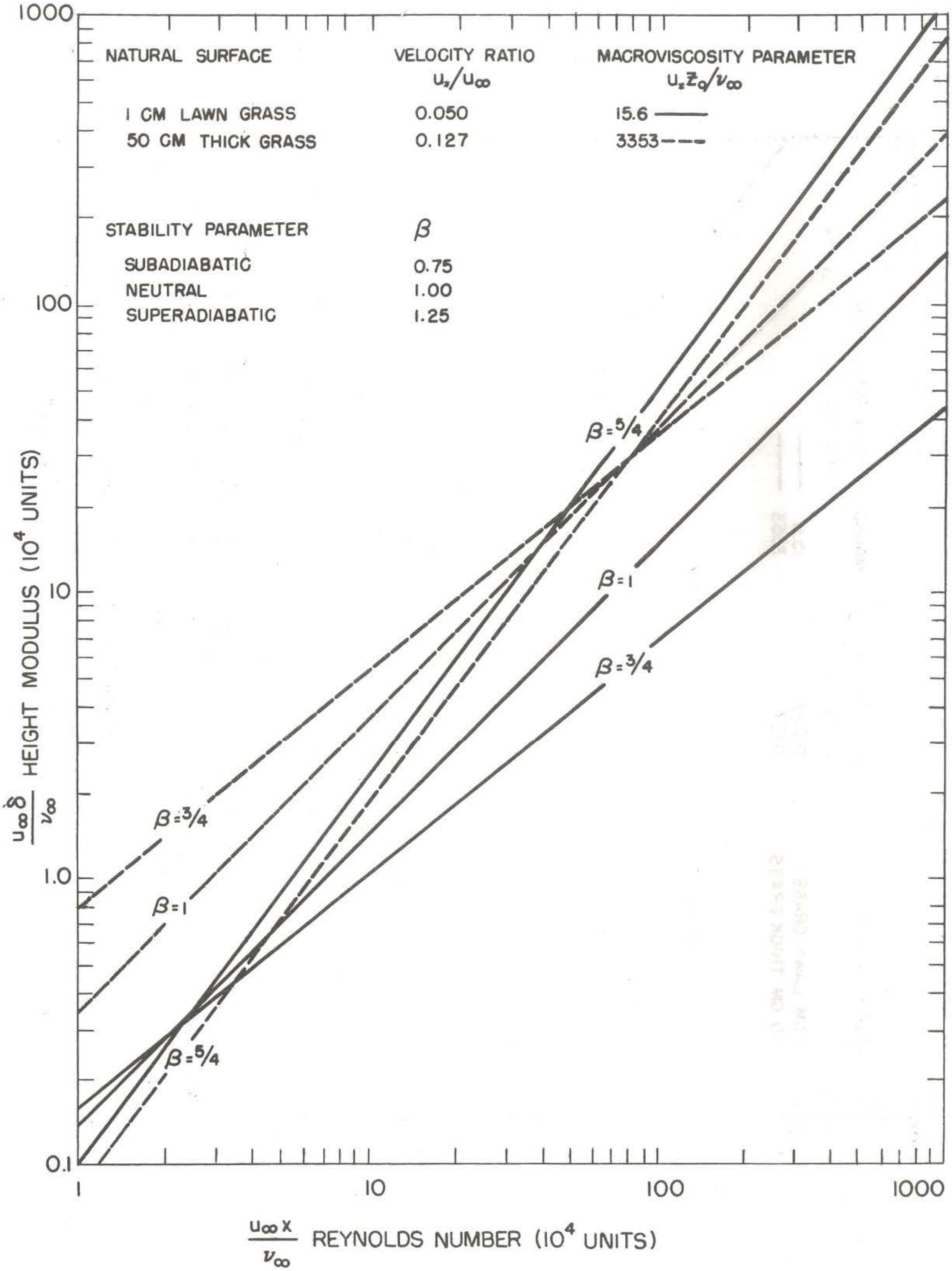

Figure 10.--Height of heated zone as function of velocity ratio, macroviscosity parameter, and stability parameter

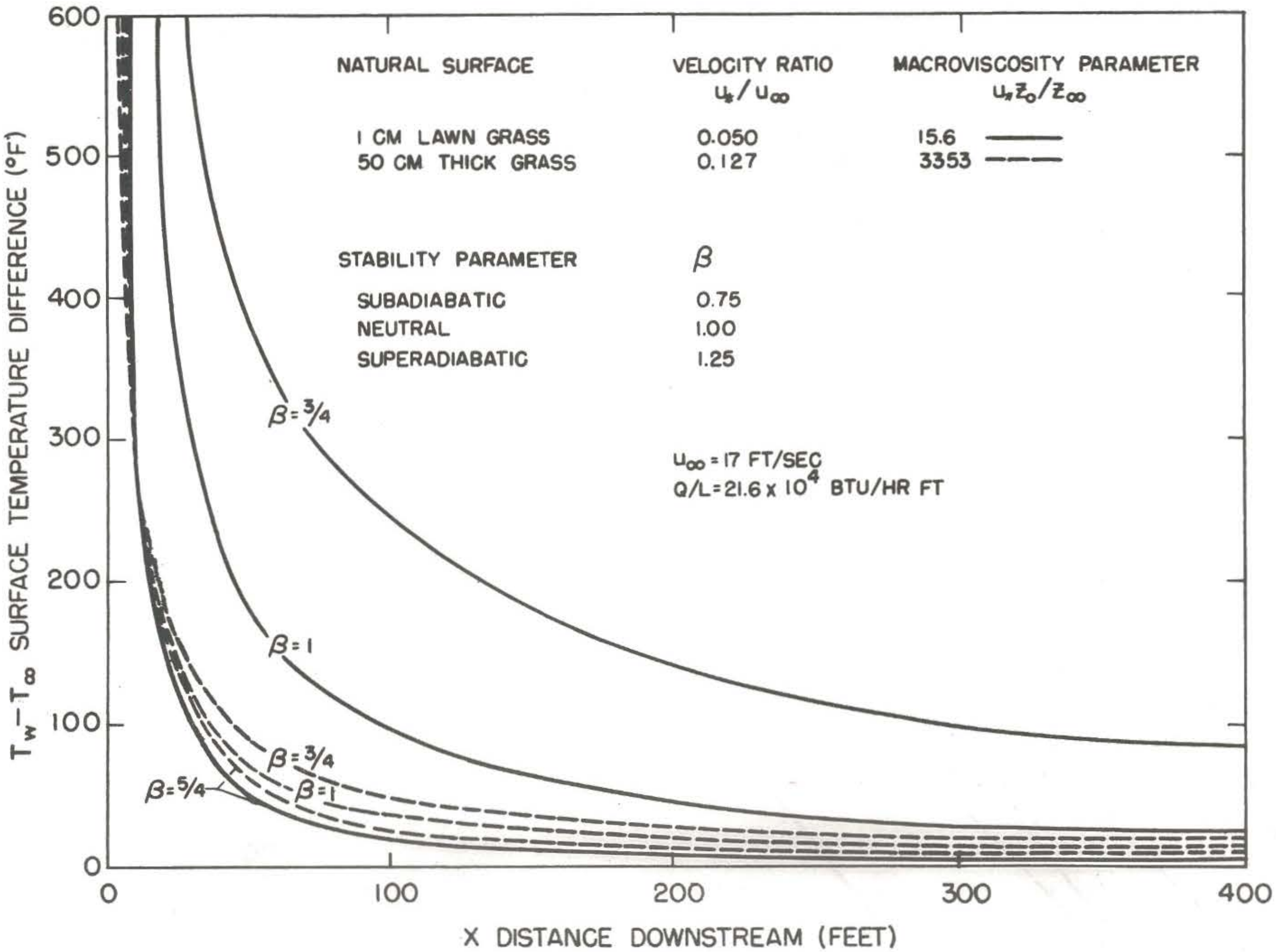

Figure 11.--Decay of surface temperature as function of velocity ratio, macroviscosity parameter, and stability parameter

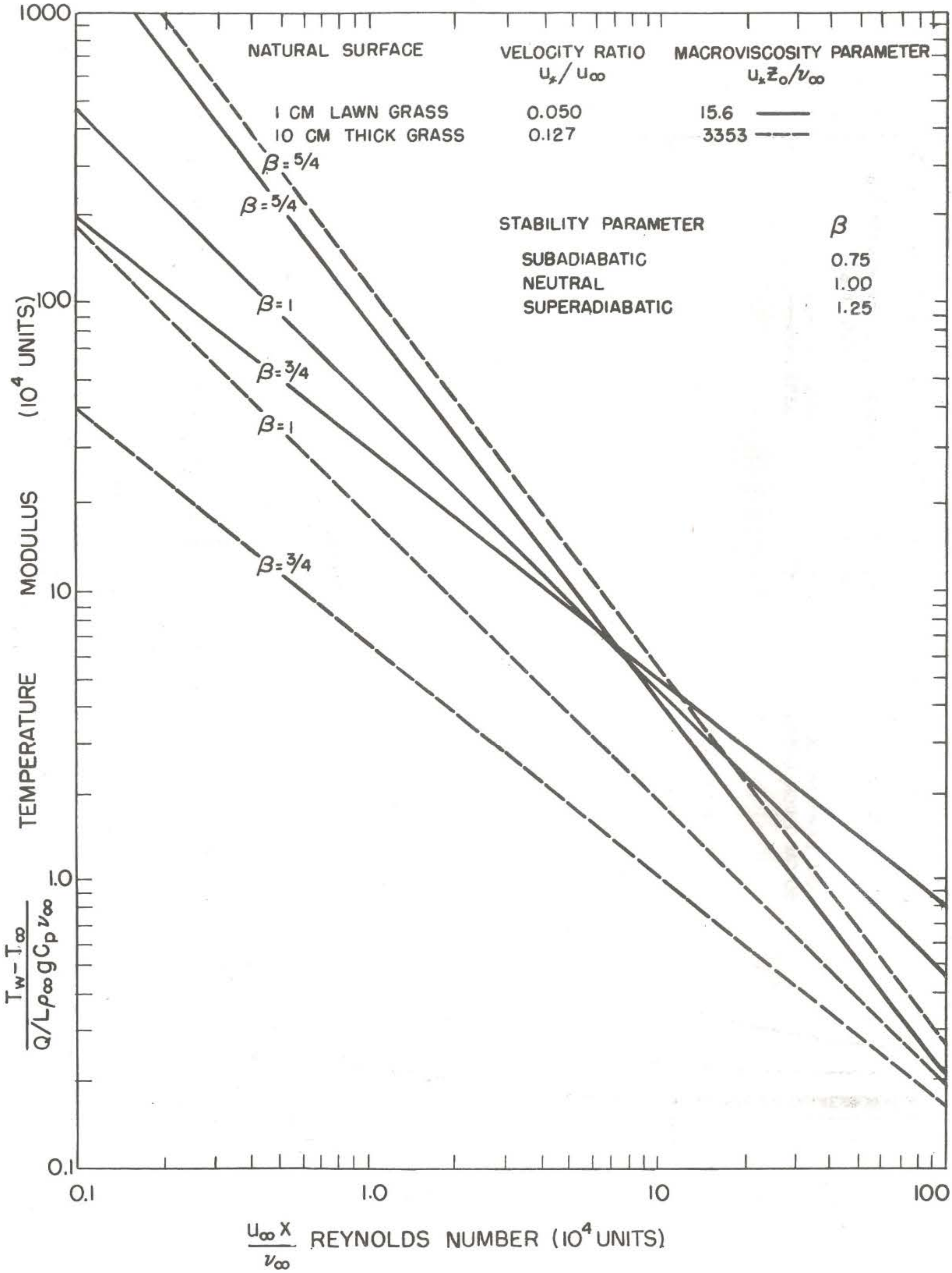

Figure 12.--Decay of temperature as function of velocity ratio, macroviscosity parameter, and stability parameter

Decay of temperature downstream from the heat source may be calculated in terms of the fuel evaporation rate M which represents the actual rate at which fuel is consumed. This rate does not represent radiative or convective energy rates directly since the efficiency of combustion and partition of energy are unknown. By a heat balance there results

$$\frac{Q}{L} = ML^{*}eH$$

where L^{*} is the width of the flame front, e is the combustion efficiency for forced convection, and H is the heat of combustion of the fuel. Equation (41) becomes

$$T-T_{\infty} = \frac{g(0)}{k^{\frac{1}{2-\beta}}} \left(\frac{ML^{*}eH}{\rho_{\infty} g c_p \nu_{\infty}}\right)\left(\frac{u_*}{u_{\infty}}\right)^{-\frac{\beta}{2-\beta}} \left(\frac{u_* z_o}{\nu_{\infty}}\right)^{-\left(\frac{1-\beta}{2-\beta}\right)} \left(\frac{u_{\infty} x}{\nu_{\infty}}\right)^{-\frac{1}{2-\beta}} G(\eta) \qquad (42)$$

This expression relates the heat transfer to the combustion process. The fuel evaporation rate (weight rate per unit area) and the percentage of this weight rate that is converted into sensible heat for transfer by forced convection are required.

Shear Stress and Drag Coefficient

Defining the shear stress as

$$\tau = \rho_{\infty} K_m(z) \frac{\partial u}{\partial z} = \rho_{\infty} k z_o u_* \left(\frac{z}{z_o}\right)^{\beta} \frac{\partial u}{\partial z} \qquad (43)$$

yields the drag coefficient as

$$C_D = \frac{\tau_o}{\frac{1}{2}\rho_{\infty} u_{\infty}^2} = 2kz_o \left(\frac{u_*}{u_{\infty}}\right)\left(\frac{z}{z_o}\right)^{\beta} \left.\frac{\partial (u/u_{\infty})}{\partial z}\right|_{z \to o} \qquad (44)$$

applying the transformation there results

$$C_D = \frac{\tau_o}{\frac{1}{2}\rho_{\infty} u_{\infty}^2} = 2k^{\frac{1}{2-\beta}} \left(\frac{u_*}{u_{\infty}}\right)^{\frac{\beta}{2-\beta}} \left(\frac{u_* z_o}{\nu_{\infty}}\right)^{\frac{1-\beta}{2-\beta}} \left(\frac{u_{\infty} x}{\nu_{\infty}}\right)^{-\left(\frac{1-\beta}{2-\beta}\right)} K, \qquad (45)$$

Using the definition of the friction velocity, the expression may be presented in general form as

$$C_D = \frac{\tau_o}{\frac{1}{2}\rho_\infty u_\infty^2} = 2k^{\frac{2}{2-\beta}}\left(\frac{u_*}{u_\infty}\right)^{\frac{4\beta-4}{2-\beta}}\left(\frac{u_* z_o}{\nu_\infty}\right)^{\frac{2-2\beta}{2-\beta}}\left(\frac{u_\infty x}{\nu_\infty}\right)^{-\left(\frac{2-2\beta}{2-\beta}\right)} K_1^{\,2} \tag{46}$$

Values of K_1 for the three selected cases of β are given in table 4 and presented in figure 13.

Table 4.--Variation of K_1 with β

β	K_1
0.75	0.242
1.00	0.107
1.25	0.047

For the case of neutral stability $(\beta = 1)$, the drag coefficient is

$$C_D = \frac{\tau_o}{\frac{1}{2}\rho_\infty u_\infty^2} = 0.0036 \tag{47}$$

This result states that the drag coefficient is independent of wind velocity, position, and type of natural surface. For cases of stability not equal to unity, the drag coefficient is dependent on wind velocity, position, and type of natural surface. The shear stress at the surface is

$$\begin{aligned} \tau_o &\sim u_\infty^{8/5} & \beta &= 0.75 \\ \tau_o &= 0.0018\,\rho_\infty u_\infty^2 & \beta &= 1.00 \\ \tau_o &\sim u_\infty^{8/3} & \beta &= 1.25 \end{aligned} \tag{48}$$

Theoretical results of the shear stress are presented in figure 14 for natural surfaces of lawn grass up to 1 cm high and thick grass up to 50 cm high. The downstream position for the cases of stability unequal to unity was arbitrarily selected as 100 ft. Experimental values of the shear stress as determined by Vehrencamp (8) are indicated.

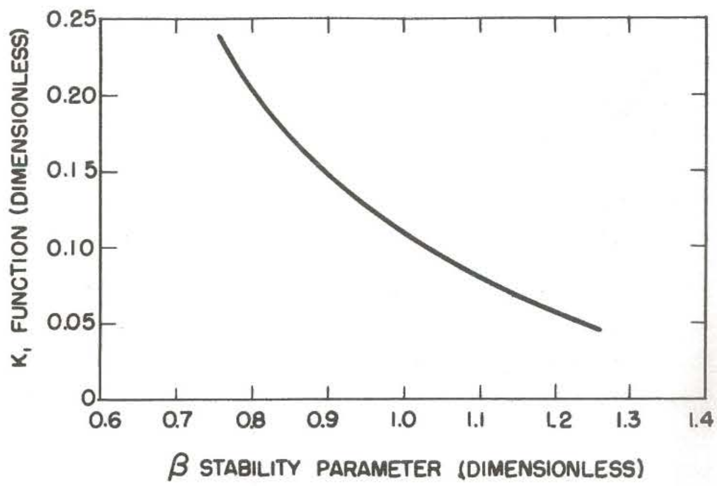

Figure 13.--Variation of function K_1 with stability

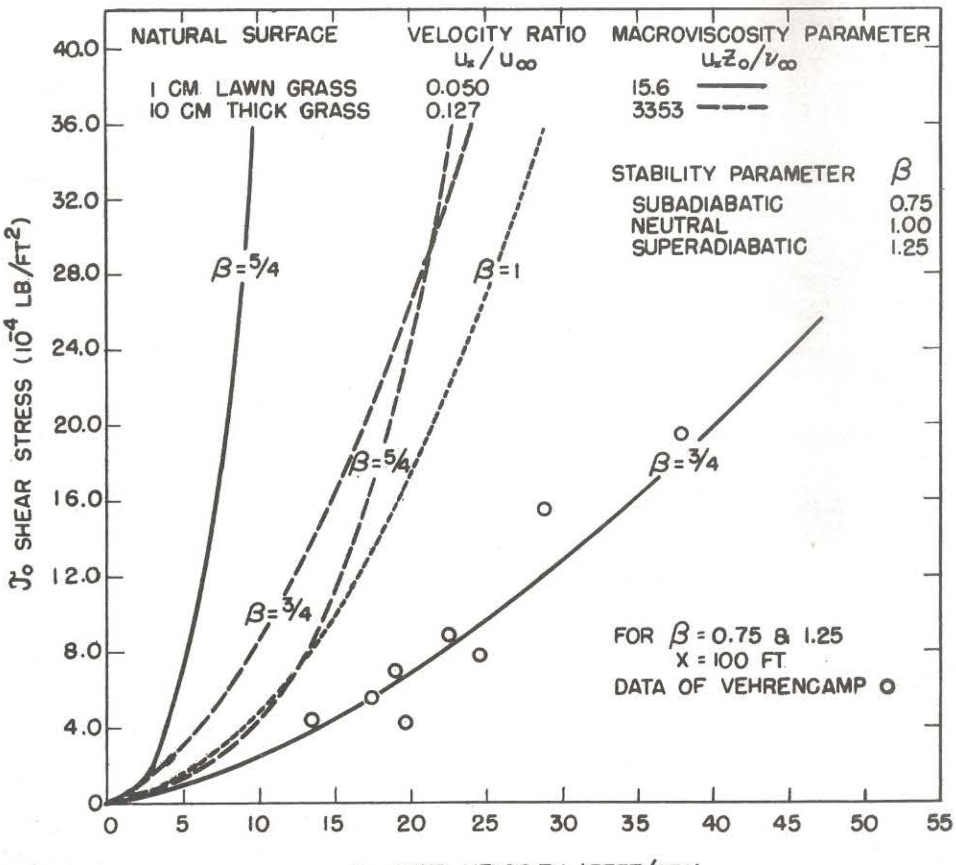

Figure 14.--Shear stress as a function of velocity (β = 1 independent of velocity ratio and macroviscosity parameter)

DISCUSSION

A mathematical theory has investigated forced convection heat transfer from a combustion process in the atmosphere. From this theory no discussion can be made of the influence of the atmospheric variables on the combustion rate, that is the buildup or decay of a fire. A discussion can be presented concerning the factors which influence the transfer of heat from a combustion zone considering only steady state forced convection.

The initial result of the investigation is the condition which must be fulfilled so that the buoyancy effect may be neglected. Physically, this is an energy relationship and is expressed as

$$\frac{Q}{Lc_p T_\infty} \ll \rho_\infty u_\infty^3$$

This condition states that the energy rate per unit area of the combustion process must be much less than the kinetic energy rate per unit area of the existing velocity field. For a given combustion zone the wind velocity must be greater than a specified magnitude so that the heat generated will be transferred by forced convection. The case of free convection only has been treated in reference (4).

Theory was developed considering three cases of atmospheric stability. Strong subadiabatic, neutral, and strong superadiabatic conditions were selected. Variation of the stability parameter taken from Deacon (2) is shown in figure 7. Values of β of 0.75 and 1.25 represent extreme cases.

The velocity ratio and macroviscosity parameter are determined by the type of natural surface. Further discussion of these quantities is presented in references (6,7,8). Representative values for several natural surfaces are presented in table 2. These parameters for other types of natural surfaces such as brush types, orchard groves, forested areas, and urban structure classes are presently unknown. At the present time these parameters are believed to be constants for any given surface. Throughout this study two natural surfaces of 1 cm lawn grass and 50 cm thick grass are compared for the three cases of stability. The very smooth surface of table 2 was not considered since values of $\frac{u_* z_o}{\nu_\infty} > 2.5$ are representative of aerodynamically rough surfaces. On this basis growth of the heated region and decay of surface temperature is presented for an arbitrarily selected wind velocity and heat output.

The superadiabatic condition extends the heated zone to the greatest height and consequently has a more rapid decay of temperature. Thus for a given combustion process maximum vertical temperature penetration would occur for this condition. A subadiabatic condition experiences

a very shallow heated region with a slow temperature decay. This condition would experience a more rapid drying out of the surface fuel downstream from the combustion zone since the heat generated would be concentrated near the surface.

As the surface roughness is increased the importance of stability is decreased. The heated zones and temperature patterns tend to approach each other. The surface parameters were plotted in figure 15. Extrapolating to aerodynamically rougher surface, values of $u_*/u_\infty = 0.22$ and $u_* z_o/\nu_\infty = 10^5$ were selected. The heated zone was compared for a wind velocity of 17 ft/sec and is presented in table 5.

Table 5.--Growth of heated zone

$u_*/u_o = 0.22 \quad u_* z_o/\nu_\infty = 10^5 \quad u_\infty = 17$ ft/sec

x	δ Ft		
Ft	β = 0.75	β = 1.00	β = 1.25
50	31.4	34.8	24.6
100	54.5	59.0	60.0
200	95.2	128.0	142.0
300	130.0	187.0	260.0
400	164.0	256.0	390.0

Assuming the validity of the extrapolation, atmospheric stability does not markedly influence the growth of the heated region for very rough surfaces.

Influence of stability and surface roughness on the aerodynamic drag is of considerable interest. Measurements of the shear stress at the surface have been made by Sheppard (6) and Vehrencamp (8). Theory presented in this study shows that for neutral stability the drag coefficient is constant. For cases of stability parameter other than unity the drag coefficient is a function of the natural surface and Reynolds Number. The shear stress as a function of wind velocity is shown in figure 14. The case $\beta = 1$ is independent of surface type and position. For cases of $\beta =$ 0.75 and 1.25 two surface types are compared for an arbitrarily selected position of 100 ft. For the lower values of velocity ratio and macroviscosity parameter the shear stress is greatly influenced. For the aerodynamically rougher surface the effect of stability is decreased. This is consistent with the previous results regarding the growth of the heated zone and decay of surface temperature.

Shear stress results of Vehrencamp which were obtained on a smooth dry-lake bed are presented in figure 14. Theory yields the drag coefficient for neutral stability as 0.0036 which is higher by a factor of 3 when compared with 0.00125 calculated from shear-stress and velocity data by Vehrencamp. The shear stress is velocity dependent as specified by the stability parameter. This dependency is

$$\tau_o \sim u_\infty^{\frac{2}{2-\beta}}$$

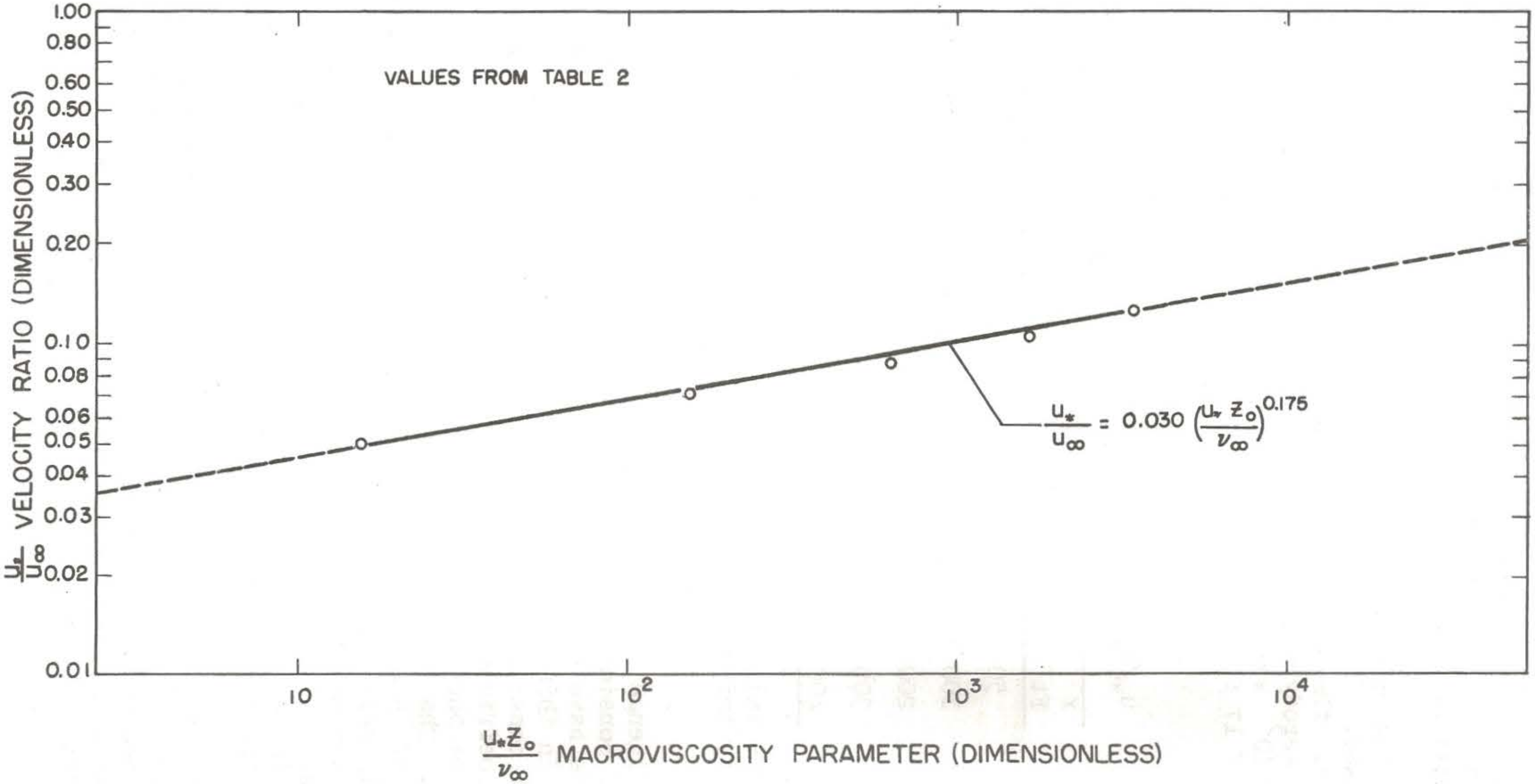

Figure 15.--Relationship between velocity ratio and macroviscosity parameter

which indicates the importance of accurate determination of the stability of the atmosphere when undertaking field experiments.

The main step in the theoretical study was the selection of the eddy diffusivity for momentum and heat applicable to transport processes within the surface air layer. If one assumes the eddy diffusivity for momentum and heat as equal and postulates this quantity to be proportional to the macroviscosity parameter and the normalized distance from the surface to a power, then

$$K_m^{(z)} = K_H^{(z)} \sim \left(\frac{u_* z_o}{\nu_\infty}\right)\left(\frac{z}{z_o}\right)^\beta$$

The foregoing expression is a generalized eddy diffusivity which is a function of the surface roughness, distance from the surface, and stability. Experimental verification has been presented by Deacon (2) and is here considered the best correlation available at present. Deacon himself, however, expressed some doubt about the applicability of his results to correlations of extreme stability or instability. This procedure is a step commonly employed by the theorist to produce an acceptable mathematical theory. It may be recalled that choosing a constant diffusion coefficient (laminar flow case) produces an unacceptable theory.

Certain interesting consequences result from theory. Since η is the independent variable given by equation (25), the velocity profiles are dependent on the type of natural surface, Reynolds Number, and stability parameter. The independent variable represents the dimensionless height and reveals the generalized correlation parameter for wind velocity in the atmosphere. As evidenced by figure 4 where $df/d\eta = u/u_\infty$, correlation of the velocity profiles are not universal but depend on the stability parameter. Likewise, temperature profiles downstream from a heat source in the atmosphere may be correlated by equation (41) in terms of the independent variable β . Again figure 5 shows that the dimensionless temperature profiles are not universal.

CONCLUSIONS

1. The kinetic energy rate per unit area of the existing velocity field must be much greater than the energy rate per unit area of the combustion process for transmission of heat by forced convection. Theory yields the condition as

$$\rho_\infty u_\infty^3 \gg \frac{Q}{Lc_p T_\infty}$$

2. Growth of the heated zone and consequently decay of the temperature profiles are markedly affected by atmospheric stability for small values of the velocity ratio and macroviscosity parameter. For very rough

surfaces, the heated zones and temperature profiles tend to merge for all cases of stability. The effect of atmospheric stability on forced convection heat transfer over very rough surfaces is unimportant.

3. The drag coefficient for neutral stability ($\beta = 1$) is independent of velocity ratio, macroviscosity parameter, and Reynolds Number. Theory yields the drag coefficient as

$$C_D = \frac{\tau_o}{\frac{1}{2}\rho_\infty u_\infty^2} = 0.0036$$

For cases of stability unequal to unity, the drag coefficient is dependent upon velocity ratio, macroviscosity parameter, and Reynolds Number.

4. The shear stress at the surface is proportional to the wind velocity, according to

$$\tau_o \sim u_\infty^{\frac{2}{2-\beta}}$$

Atmospheric stability does influence the shear stress for small values of velocity ratio and macroviscosity parameter but is not very important for very rough surfaces.

NOMENCLATURE

The following nomenclature is used in the paper:

A = constant

B = constant

c_p = specific heat at constant pressure, BTU/lb $^{\circ}F$

C_D = drag coefficient, dimensionless

e = combustion efficiency, dimensionless

$f(\eta)$ = function of independent variable defined by eq. (25), dimensionless

F = function of transformed independent variable, dimensionless

g = gravitational constant, ft/sec^2

$g(\eta)$ = function of independent variable defined by eq. (26), dimensionless

$G(\eta)$ = function of independent variable, dimensionless

H = heating value of fuel, BTU/lb.

J = mechanical heat equivalent, ft lb/BTU

k = Karman constant, 0.40

k' = thermal conductivity of air, BTU/sec sq ft $^{\circ}F$/ft

K = constant, equation (32)

K_1 = constant, equation (35)

$K_H(z)$ = eddy diffusivity for heat, ft^2/sec

$K_m(z)$ = eddy diffusivity for momentum, ft^2/sec

L = line source length, ft

L^* = flame front width, ft

M = fuel evaporation rate, lb/ft^2 sec

N = transformed independent variable, dimensionless

p = pressure in source zone, psf

p_∞ = atmospheric pressure, psf

P = dimensionless pressure

Q = heat output of source, BTU/sec

R = forest fuel rate, tons/acre min.

T = temperature in source zone, ^{o}R

T_∞ = ambient temperature, ^{o}R

T_w = surface temperature, ^{o}R

u = velocity component in horizontal direction, fps

u_∞ = wind velocity, fps

u_* = friction velocity, fps

U = dimensionless velocity

w = velocity component in vertical direction, fps

W = dimensionless velocity

x = horizontal distance from source, ft

x_o = reference length, ft

X = dimensionless distance

z = vertical distance above surface, ft

z_o = roughness length, ft

Z = dimensionless distance

Re = Reynolds Number, dimensionless

Ri = Richardson Number, dimensionless

β = stability parameter, dimensionless

δ = height of heated zone, ft

η = independent variable, dimensionless

η_δ = independent variable at edge of heated zone, dimensionless

μ = absolute viscosity, lb sec/ft^2

ν_∞ = ambient kinematic viscosity, ft^2/sec

ρ = mass density of source zone, lb sec^2/ft^4

ρ_∞ = ambient mass density, lb sec^2/ft^4

σ = Prandtl Number, dimensionless

τ = shear stress in source zone, lb/ft^2

τ_o = surface shear stress, lb/ft^2

ψ = stream function, ft^2/sec

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